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Texts choice and remixing: Giovanni Toffoli and Stefano Lariccia – Sapienza Università di Roma, DigiLab and LINK srl
Content revision and English translation: Ingrid Barth – Tel Aviv University (TAU), Division of Foreign Languages.
Most texts are inspired by: "Educazione al pensiero critico" (Editoriale Scientifica) by Francesco Piro – Università degli studi di Salerno, Dipartimento di Scienze Umane, Filosofiche e della Formazione.
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UNIT 3 – Reasoning with propositional calculus

One of the main objectives of the study of reasoning is to help you get used to critical reading; to be able to extract the information contained in a text and to distinguish, for example, what information is explicitly given and which is given in implicit form; which inferences the writer intends to suggest to the reader and which he can however do on his own initiative.

Deductive reasoning

Deduction is a process of inference that leads to a certain conclusion from premises that fully justify it: given the premises, there is no other possible conclusion.

Kids find it boring to do their homework. So my niece Emma, who is a little girl, finds it boring.

This argument refers to a premise that you might not agree with but that for whom says it seems to represent an unquestionable truth; anyway, if we take it as an example of deduction, the only thing that interests us is this: that the conclusion does not add new information but only makes explicit that already contained (implicitly) in the premises; the result of a deduction is literally a *tautology*.

So, what's the point of deduction? In itself, it does not allow you to reach new truths, but "only" to preserve the truths that we already possess; and that is no small thing. The correct use of deduction constitutes a kind of hygiene of the mind, the simplest that exists. This, to use a typical expression of logic, constitutes a *necessary but not sufficient condition* to make good reasoning.

Deductive reasoning was among the first that has been tried to formalize, since the antiquity of classical Greece. Its study has made progress in the modern and contemporary age, also thanks to an attempt to "mathematize" the logic, which perhaps began with Leibniz and continued in recent centuries with scholars such as Boole, Frege, Peano and Russel.

Propositional calculus and its limits

In formal logic, the term "proposition" is mostly used as a synonym for "statement", because only those propositions are taken into consideration for which it makes sense to ask whether they are true or false. This is called the *principle of bivalence*; the term must not deceive: bivalent does not mean ambiguous or ambivalent; on the contrary, it means that no middle ground between true and false is taken into consideration.

Propositional calculus, or *calculus of propositions,* is a symbolic calculation where we are only interested in

- *propositions*, represented by *propositional variables*
- the way in which several propositions can be *composed*

- the way in which the *truth value* of complex propositions is *derived*.

Propositional calculus leaves to other disciplines to investigate the concept of truth, that is the criteria for deciding how to assign a truth value to an elementary statement; these could include plausibility, consent, observation, tradition, aso. Propositional calculus analyses only the *rational* mechanisms that preserve the truth when composing elementary statements into complex statements; who, for example, believes, or anyway accepts as premises in a reasoning

Allah is great. Muhammad is the prophet of Allah.

should not have difficulty accepting as a logical conclusion

Allah is great <u>and</u> Muhammad is his prophet.

The principle of bivalence (every proposition can be either true or false) excludes every intermediate nuance, i.e. the range of truth values suggested by everyday language expressions such as "maybe ...", "probably ...", "I'm sure ...". Moreover, only discourses that have a "logical" structure are taken into consideration, that is, in which the component propositions are linked by a restricted set of logical connectives, also called *logical operators*.

The logical connectives of the propositional calculus

The only logical connectives allowed by the propositional calculus, to compose complex propositions, are those that correspond, in English, to the adverb "not", to the conjunctions "and" and "or", and to *conditional* expressions of the type "if ... then ...".

An overall view

For convenience, we present now a synoptic table that outlines a correspondence between the logical connectives used in the formalization of the propositional calculus and the English words that often recall them, also introducing the symbols that can be used in the *formulas* of the propositional calculus, that is, in expressions composed of propositional variables and logical connectives.

English word or expression	arguments number	operation name	preferred symbol	alternate symbols	set operation	latin symbol	boolean symbol
not	1	negation	¬	~	–	non	NOT
and	2	conjunction	∧	&	∩	et	AND
or	2	(inclusive) disjunction	∨		∪	vel	OR
“	2	exclusive disjunction	⊕	/ ⊻		aut-aut	XOR
“	2	incompatibility disjunction	|				NAND
if ... then ...	2	implication	→	⇒			
If and only if ... then ...	2	double implication	↔	⇔			

For each connective we will give in the following, through a specific *truth table*, an interpretation in terms of logical operation. The truth table for a logical connective enumerates the possible combinations of the truth values of the arguments (1 or 2 in the simplest cases) and, for each combination, the truth value of the compound proposition obtained by linking the arguments with that connective.

We used a lighter shade of grey for the two lines of the table marked with the operation names *exclusive disjunction* and *incompatibility disjunction*, to indicate that we could have omitted them: they are not necessary in constructing the formulas of the propositional calculus, but are only useful to make some formulas and some derivations more concise and readable. In fact, by expressing ourselves with the terminology of Boolean algebra, it is always possible to rewrite a formula replacing the XOR and NAND operators with the *fundamental operators* NOT, AND, and NOT.

The word "and" as a logical conjunction

Rome is the capital of Italy and Paris is the capital of France.

The operation represented by the English conjunction "and" is called *conjunction* not only in the language of grammar but also in that of logic. One way to formalize its meaning, that is, to express it in a clear and incontrovertible way, is to use a truth table.

p	q	p ∧ q
true	true	true
true	false	false
false	true	false
false	false	false

This table, in its first two columns, enumerates all possible combinations of the truth values of two propositions and in the third column it reports the truth value of the proposition "p and q", which is obtained as their *conjunction*, that is, the truth value of a proposition that affirmed both of them.

Some remarks:

- to avoid confusion, instead of the word "and", we used the symbol "∧" (which also reads "and") as a connective, i.e. to represent the conjunction operator; "∧" has the advantage of being similar to the operator "⋂" used for set intersection, which has some analogy with the logical conjunction; another symbol used is Latin "et", which can also be written as "&"
- to indicate that we are talking about propositions in general, not specific statements, we have used the symbols *p* and *q*, since it is common to use the letters *p*, *q*, *r*, ... as propositional variables
- we represented the two truth values with the pair of English words (*true*, *false*), but we could have used other pairs, such as (*True*, *False*) or (*T*, *F*) or (*1*, *0*).

If the variable *p* stood for the proposition "Rome is the capital of Italy" and *q* for "Paris is the capital of France", and we considered both things true, by using the first line of the truth table we would get that the complex proposition "Rome is the capital of Italy and Paris is the capital of France" is also true. But the role of the truth table is to predict all possible combinations of the truth values of "*p*" and "*q*", not just this one.

The thief saw the policeman and [the thief] escaped.

Somebody could guess that the two joint propositions above are not "logically" independent, that the authentic interpretation of this complex statement is that the thief escaped *after* he saw the policeman, or also *because* he saw the policeman. But, if we stick to an interpretation in which the English conjunction "and" is equivalent to the operator of the logical conjunction "∧", we can use only the truth table, after having ascertained whether it is true that the thief has faced the policeman and whether it is true that the thief has escaped.

The word "not" as a negation

Roma is not the capital of Italy.

The operator represented by the word "not", which applies to a single proposition, results in one whose truth value is inverted with respect to the original one; that is, it transforms a true proposition into a false one and vice versa. The truth table is very simple

p	¬ p
true	false
false	true

We used the "¬" symbol (which reads "not") to represent the logical operator of negation.

We can add here that the propositional calculus accepts in itself the *principle of non-contradiction*, whereby if from certain premises one could derive, for a proposition *p*, both that it is true and that it is false, then at least one of the premises is necessarily false. It is an *axiom*, that is, a rule of derivation that can be used as a theorem but that we believe we do not have to prove.

The word "or" as a logical disjunction

The formal interpretation of the word "or" is more problematic than that of the word "and" [3, 4].

(o.1) *Paul is very intelligent or has studied a lot.*

In this example we talk about a certain Paul who has just passed an exam.

The truth table for such a proposition could be the following:

p	q	p V q
true	true	true
true	false	true
false	true	true
false	false	false

A single case leads to a false result; in fact, for the complex proposition to be true, it suffices that one of the two component propositions is, without excluding that both are. For inclusive logical disjunction ("or" inclusive) we have used the symbol "V" (which reads "or") as connective; "V" has the advantage of being similar to the "U" operator used for the *union* of sets, which has some analogy with the logical disjunction; another symbol used is Latin "vel".

The exclusive disjunction

(o.2) *At breakfast Marta [or] drinks a coffee or [drinks] a tea*

The truth table for a proposition of this second type is a bit different from the previous one:

p	q	p ⊕ q
true	true	false

true	false	true
false	true	true
false	false	false

In other words, in this which is called *exclusive disjunction* (exclusive "or"), a new case is considered false, one in which both simple propositions are true. The exclusive "or" operator, in addition to the "⊕" symbol used in the table, may be represented by the pseudo-English XOR (eXclusive OR) or the Latin "aut-aut"

The incompatibility disjunction

(o.3) *at the table you eat or talk*

(o.3-bis) *at the table you either eat or talk*

In this third case, called *incompatibility*, the symbol used by logicians is the vertical bar "|". The truth table is still different from the previous ones, because the only possibility that is excluded is that in which both simple propositions are true: eating and talking are considered incompatible actions. Instead, nothing prevents us from fasting and silence (last line).

p	q	p | q
true	true	false
true	false	true
false	true	true
false	false	true

For the incompatibility disjunction the NAND symbol is also used; it is the contraction of "NOT AND", which we can express in words like "it is false that they are both true". In fact, the last column is obtained by inverting that of the truth table of the AND operator (conjunction "and").

The examples of disjunction that we have presented make us understand that the formalization with the logical connectives of a sentence in natural language is not so obvious but requires an effort of interpretation: usually the proposed formalization appears obvious only if we share some assumptions of context. For example:

- in (o.1) it is probably assumed that the exam was difficult
- in (o.2) it is taken into account that it is not common to take both coffee and tea for breakfast
- in (o.3) it is assumed that a rule of good manners is respected; in reality, rather than an *apophantic* sentence (which shows how things are), it could be considered a *prescriptive* statement, which enunciates a norm to be followed, and of which it cannot be said whether it is true or false.

Reasoning with the propositional calculus

How to use logical connectives in reasoning

Deferring to a subsequent unit the examination of the two most difficult logical connectives - the implication and the double implication -, we see here, for those already defined, which operations can be performed on a formula that contains them [3]. Until we do this, it cannot be said that the propositional calculus serves us to make arguments.
Also in this case we use a synoptic table, which looks rather complex but gives us a convenient overall view.

connettive	*operation code*	*operation name*	*left formula(s)*	*proves*	*right formula*	*in words*

Negation ¬	DNI	Double Negation Introduction	p	$\vdash$	$\neg\neg p$	*a double negation states*
	DNE	Double Negation Elimination	$\neg\neg p$	$\vdash$	p	
Conjunction &	&I	Conjunction Introduction	p, q	$\vdash$	$p \& q$	*if two statements are true individually, so is their conjunction*
	&E	Conjunction Elimination	$p \& q$	$\vdash$	p, q	*if the conjunction of two propositions is true, so is each of them taken individually*
Disjunction V	VI	Disjunction Introduction	p	$\vdash$	$p \vee q$	*if a statement is true, it is also true in disjunction with any other statement*
	VE	Disjunction Elimination	$p \vee q, \neg q$ $p \vee q, \neg p$	$\vdash$ $\vdash$	p q	*if the disjunction between a statement p and a statement q is true, and if it is known that the second is false, then the first is true, and vice versa*

Admissible truth-preserving rewrite operations

Deductive arguments as a paraphrase

The "$\vdash$" symbol, which we have not yet met, stands for "proves"; it has a meaning similar to the connective "→", which stands for "implies"; but there is a significant difference:

- "→" is a connective that operates within the logic of propositions, i.e. it serves to construct complex propositions and, like the other connectives-operators, has its own truth table
- "$\vdash$" is part of the logical-mathematical *metalanguage* that we need here to talk about propositional logic and to say what is permissible within it; more precisely, to say how one can rewrite his formulas (*re-formulate* them) in order to make explicit hidden truths;

referring to natural language, we would say that these operations serve to make *paraphrases*.

For each of the formula-rewriting operations described in the table, the fact that it preserves the truth value of the left formula can be demonstrated on the basis of the assumptions of the propositional calculus; that is, it constitutes a theorem. We could prove it by calculating the truth tables of the left-hand and right-hand formulas, starting from those of the connectives used, but we will do without them.

Examples of application of the rewrite operations

Rewriting operations based on the properties of logical connectives allow you to demonstrate the validity of derivations that could be written as follows in English:

(DNE) *It is false that I cannot sing*. $\vdash$ *I can sing*.

(&I) *Mary can play*. *Mary can sing*. $\vdash$ *Mary can play and sing*.

(&E) *Mary can play and sing*. $\vdash$ *Mary can play*. *Mary can sing*.

(VI) *Arsenal won*. $\vdash$ *Arsenal won or Chelsea tied*.

(VE) *Paula is tall or wearing heels. Paula does not wear heels*. ├ *Paula is tall*.

Reading and understanding

One of the main objectives of the study of reasoning is to help you get used to critical reading; to be able to extract the information contained in a text and to distinguish, for example, what information is explicitly given and which is given in implicit form; which inferences the writer intends to suggest to the reader and which he can however do on his own initiative.

Some basic techniques

In another learning path, dedicated to the basic techniques for the analysis of texts - techniques that can even be automated within certain limits -, we have taken into consideration, among others, the following types of processing:

- the *segmentation* of the text into sentences, for which we mainly consider punctuation marks and possibly also the arrangement of the text on the page or on the screen.
- *POS-tagging*, i.e. the classification of words into *parts of speech* (POS) or grammatical categories.
- the extraction of the most significant *fragments* of the sentence, mostly corresponding to *nominal chunks* and *verbal chunks.*
- the reconstruction of the complete structure of the text through the traditional syntactic analysis or *constituent analysis*, which allows to derive a *syntactic tree*, in which the root represents an entire sentence and each node an intermediate constituent (verbal, nominal, prepositional phrases, etc.)
- *dependence analysis*, i.e. the reconstruction of the logical structure of the text in terms of links between words; it requires a thorough knowledge of the lexicon: not only of *grammatical* words (especially conjunctions and prepositions), but also of *lexical* words - such as verbs and nouns - each of which is characterized by its *valence*, that is, by the number and type of links with other elements of the text (such as complements) that must or can complete their meaning.

Rewording the text

As a first step towards understanding a text, you could try to rewrite it in a simplified form: for example, the following text, reported by [6]

> *Prime Minister Putin, the country's paramount leader, cut short a trip to Siberia, returning to Moscow to oversee the government's response. Mr. Putin built his reputation in part on his success at suppressing terrorism, so the attacks could be considered a challenge to his stature*

could be rewritten as follows:

> *Prime Minister Putin cut short a trip to Siberia.*
> *Prime Minister Putin was the country's paramount leader.*
> *Prime Minister Putin returned to Moscow to oversee the government's response.*
> *Mr. Putin built his reputation in part on his success at suppressing terrorism.*
> *The attacks could be considered a challenge to his stature.*

What have we done? Essentially, we have rewritten the subordinate sentences as independent sentences and we have made explicit the *co-referentiality* - that is the fact that most of the information refers to the same individual - by repeating the name of this until boredom. If it had been useful, we could also have broken coordinated sentences and replaced difficult words with others of more common use or with paraphrases.

In the end we got a set of declarative sentences, or *propositions*, that, if we believe the author of the text, we could consider all true; they could allow us to make a series of inferences, if we also had some general premises, like *the leader of a country shortens a journey if and only if he is very worried*.

This rewriting and simplification operation is presented in [6] as a step preliminary to the design of a set of questions to verify the understanding of the text itself, such as

Who shortened a trip to Siberia?
How Putin built his reputation?

BIBLIOGRAPHY AND WEBOGRAPHY

[1] Willy - Liceo Alfano I (Salerno), Breve introduzione alla logica, 2016

http://www.liceoalfano1.gov.it/documenti/category/64-alfano-logico.html?download=375

[2] Francesco Piro, *L'argomentazione - Invito al pensiero e alla lettura critica*, 2015, Out of business – Handout for educational use (First part: *tipi di enunciato, tipi di ragionamento* - pages 1-25)

http://www.liceoalfano1.gov.it/documenti/category/64-alfano-logico.html?download=316

[3] Francesco Piro, *Manuale di educazione al pensiero critico. Comprendere e argomentare*, foreword by Tullio De Mauro, Editoriale Scientifica, Napoli, 2015, pp.280

[4] CIDI - Centro di iniziativa democratica degli insegnanti (editor), *Introduzione alla logica*, Editori Riuniti, 1976

[5] Michael Heilman, Extracting Simplified Statements for Factual Question Generation

http://www.cs.cmu.edu/~mheilman/papers/heilman-smith-qg-extr-facts.ppt