

“Up to University” EU funded Project (**Up2U**) – “University as a Hub” sub-project (**U-Hub**)

Learning Path: “**Education towards Critical Thinking**” (ECT)

Texts choice and remixing: Giovanni Toffoli and Stefano Lariccia – Sapienza Università di Roma, DigiLab and LINK srl

Content revision and English translation: Ingrid Barth – Tel Aviv University (TAU), Division of Foreign Languages.

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UNIT 4.1 – Implication and hypothetical reasoning

Implication and double Implication

As we mentioned in the previous unit, we obtain *conditional propositions* when using one of the two connectives “ $\rightarrow$ ” and “ $\leftrightarrow$ ”, which represent the *implication* and the *double implication* respectively. Both operations bind two propositions, of which the one on the left is called the *antecedent* and the one on the right is called the *consequent*. In the case of double implication, we speak of *biconditional propositions*.

The expressions “if ... then ...” and “if and only if ... then ...”

The terms “antecedent” and “consequent” are to be understood in a logical sense and not in a temporal one. Often, in Italian, the simple implication (SI) is indicated by a sentence such as “if ... then ...”, in which “if” introduces the antecedent and “then” introduces the consequent; in the double implication (DI), instead of “if” it could be “if and only if”. But the antecedent does not always precede the consequent in a discourse that contains a hypothetical reasoning.

Examples:

(SI) *If it rains [then] the road is wet*

(DI) *If and only if the triangle T has all equal angles [then the triangle T] is equilateral*

Note that the road may also be wet for other reasons, for example because it has been watered for cleaning purposes; the second example (double implication) is not a *tautology*, because having the same angles is not part of the definition of an equilateral triangle: it is a true theorem but to be proven.

The truth tables of the simple implication and double implication operators are, respectively

p	q	$p \rightarrow q$		p	q	$p \leftrightarrow q$
true	true	true		true	true	true
true	false	false	and	true	false	false
false	true	true		false	true	false
false	false	true		false	false	true

The proposition " $p \rightarrow q$ " reads "*p implies q*", while " $p \leftrightarrow q$ " reads "*p strictly implies q*" or "*p is logically equivalent to q*".

The second table tells us that a double implication is only true if the antecedent and the consequent are both true or both false: for this reason, the double implication is also called *logical equivalence*:

(DI) $p \leftrightarrow q$ is true [only] if the truth values of antecedent and consequent are the same

More questionable may appear the truth table of the simple implication:

(SI) $p \rightarrow q$ is false [only] if the antecedent is true and the consequent is false

The first two lines should not surprise us, since, even in the informal language, "A implies B" means "if A is true, then B must also be true and cannot be false". The last two lines, however, tell us that the simple implication, as a complex proposition, is always considered true if the antecedent is false; perhaps it would be better to remove those two lines completely from the table, recognizing that the implication is a logical operation "significant" (interesting, useful) only in the case of a true antecedent. The prevailing convention is instead to affirm, with the medieval logicians, that "ex falso sequitur quodlibet", that is, that "from a fake antecedent you can derive what you want"; it is a bit like if we said, with a certain less refined expression, "if my grandfather had the wheels it would be a wheelbarrow".

In English, hypothetical sentences can use conjunctions or adverbs other than "if" and "then", and often bring with them the inversion of the order of antecedent and consequent and the replacement of a subordinate sentence with a coordinated or an independent one. The two examples above, could be reformulated as follows:

(SI) *The road is [always] wet in case of rain*

(DI) *Triangle T is equilateral because all its angles are equal; it would not be equilateral if it did not have all the equal angles*

The operations allowed by implication

As in the case of the other logical connectives, implication connectives are also of interest to us because their properties, as they are defined by their truth tables, can be used within an argument. Let's see how, by examining the two rewriting operations they allow,

Affirming the antecedent

This operation allows us to replace the implication with the consequent when the antecedent is true; that is, *if the premise is true, the consequence is also true*; in formulas and with an example:

$p \rightarrow q, p \vdash q$

If it rains the road is wet

It's raining, so the road is wet

For historical reasons, the *affirming the antecedent* operation is also called *modus ponens*, a Latin term which is the name of a form of reasoning - a "syllogism" - already studied by Aristotle.

Denying the consequent

This operation allows us to replace the implication with denial of the premise when the conclusion is false. That is, *if the conclusion is false, the premise is also false*; in formulas and with an example:

$p \rightarrow q, \neg q \vdash \neg p$

If Paul and Mary are twins, [Paul and Mary] are brothers.

Paul and Mary are not brothers, so [Paul and Mary] are not twins.

For historical reasons, the *consequent negation* operation is also called *modus tollens*, a Latin term which is the name of another syllogism studied by Aristotle.

Transitivity of implication

Starting from its definition, it can be demonstrated that the conditional connective is transitive:

$(p \rightarrow q) \& (q \rightarrow r) \vdash (p \rightarrow r)$

which is that:

if p implies q and q implies r, then p implies r

Example:

If the weather is nice I go to the sea. If I go to the sea I take a bath.

therefore,

If the weather is nice, I take a bath.

Fallacies of implication-based reasoning

In another unit of this path we review the most common classes of reasoning errors, or *fallacies*. We recall here a couple of them, which depend on the misunderstanding of the properties of the logical connective of simple implication and which consist in the execution of invalid operations similar to those allowed.

Denying the antecedent

This fallacy consists in erroneously deriving the falsity of the consequence from the falsity of the premise; here is this fallacy in formulas and an example of it:

$p \rightarrow q, \neg p \vdash \neg q$

If it rains the road is wet.

It doesn't rain, then The road is not wet.

Affirming the consequent

This fallacy consists in erroneously deriving the truth of the premise from the truth of the consequence; here is this fallacy in formulas and an example of it:

$p \rightarrow q, q \vdash p$

If it rains the road is wet.

The road is wet, then It rains.

Hypothetical reasoning

Implication connectives are used to model certain dependency relationships between propositions. The operations allowed by them are used to highlight some consequences, sometimes not obvious, that can be drawn from the statements that appear in the premises; in this, the implicative connectives are similar to the other logical connectives that we have examined previously, that is, they can be included in an argument.

Instrumental use of conditional: the *conditional proof*

In reasoning, conditional expressions can also be used in an instrumental way. The use of subsidiary arguments includes *suppositional reasoning*, in which we assume something "for the sake of argument"; here is a brief presentation of this concept

one makes a supposition, draws inferences that depend upon that supposition, and then infers a conclusion that does not depend upon it [5].

within the supposition, we reason as if the supposed propositions were beliefs, using all the rules ... that have been discussed in relation to non-suppositional reasoning. [4]

An example:

If it rains, we'll go to the movies or watch the ball game on TV. But you're sick of watching the games on TV. So if it rains, we're going to the movies.

corresponds to the formula of the propositional calculus

$$p \rightarrow (q \vee r), \neg r \vdash p \rightarrow q$$

where:

as usual, the symbol $\vdash$ stands for "proves that"

the variable p corresponds to "it rains"

the variable q corresponds to "we'll go to the movies"

the variable r corresponds to "we'll watch the game on TV"

and, by kindness, we interpret the phrase "But you're sick of watching the games on TV" as the denial of r .

But we do not know if it will rain, that is, we do not know if the premise p is true; so how do we prove that the proposition $p \rightarrow q$ is true? The *conditional proof* mechanism is as follows:

(1) we make the *conjecture* that it will rain, that is we consider p true, provisionally, adding it to the list of things we know; but we take note of the fact that this constitutes a kind of debt to be repaid at the end

$$p \rightarrow (q \vee r), \neg r, p$$

(2) by applying the *Affirming the antecedent* operation to the whole of the implication and conjecture p , we obtain

$$q \vee r, \neg r$$

(3) by applying the *Disjunction elimination* operation (see the previous unit on *propositional calculus*) we obtain

$$q$$

(4) that is, having "borrowed" the truth of p (conjecture), we have shown that q is true, regardless of the truth or not of r ; now "we return the loan", writing that q is true as long as it is true p :

$$p \rightarrow q$$

Counterfactual reasoning

A *counterfactual* is a conditional whose antecedent is false or, in any case, deemed false. [6, cited in 5]

Examples:

If I had known, I would not have come.

If I had not missed the train, I would have arrived on time.

As in other cases, the grammatical structure of the antecedent and the consequent may not be made explicit in a text, but it is up to you to recognize it. For example, the two counterfactuals mentioned above could be disguised as follows:

To have known, I would not have come.

I missed the train, otherwise I would have arrived on time.

Counterfactual reasoning

is representative of the capacity, which humans possess, to abstract from certain traits of a situation that they perceive as real, to imagine alternative situations to this, to reason within the boundaries of such alternative scenarios, so obtaining information that is relevant to the real situation but that could not be directly inferred from it. [5]

Ad absurdum reasoning

A typical way of using hypothetical reasoning is the *ad absurdum* proof.

Just an example in Italian, without delaying to formalize it; to refute someone who would argue

Everything is possible.

you could argue like that:

If everything was possible, then it would be possible to prove that your statement is false. This generates a contradiction, so not everything is possible.

In the *ad absurdum* proof - in Latin *reductio ad absurdum* - instrumentally we assume the negation of the conclusion that we try to prove; from this assumption, we derive a proposition that contains a contradiction. But, due to the rule of *modus tollens* (denying the consequent), if the consequent is false - because contradictory - the antecedent must also be false, precisely that which, instrumentally, denied the conclusion that we wanted to prove.

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